E 1093 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

Fourth Semester

Core Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

(Common for Mathematics Model I, II and B.Sc. Computer Applications)

[Prior to 2013 Admissions]

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of **four** *questions has weight* 1.

I. 1 Write the equation of the plane through $P_0(x_0, y_0, z_0)$, normal to $n = Ai + Bj + Ck$.

2 Find the parametric equation for the line through the point $p(3, -4, -1)$ parallel to the vector $i + j + k$.

3 Find the unit tangent vector of the helix $r(t) = (\cos t)\, i + (\sin t)\, j + tk$.

4 What is the curvature of a straight line ?

II. 5 Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly at (1, 1).

6 Write the formula for calculating flux across a smooth closed plane curve.

7 Write the fundamental theorem of line integrals.

8 State Gauss divergence theorem.

III. 9 What is special about path independent fields ?

10 State fundamental theorem of Algebra.

11 If α, β, γ ..... be the roots of $f(x) = 0$, then find the equation whose roots are $-\alpha, -\beta, -\gamma$ .....

12 Use Descartes rule of signs to show that the equation $x^3 + 2x + 3 = 0$ has one negative and two imaginary roots.

IV. 13 In a cubic equation $x^3 + 3Hx + G = 0$ if $G = H = 0$, what we can say about the roots.

14 What is a standard reciprocal equation ?

Turn over

15 Give an example of a transcen dental function.

16 Find two numbers a and b such that a real root of $f(x) = x^3 - x - 1 = 0$ lies between a and b.

$(4 \times 1 = 4)$

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Find the point where the line $x = \frac{8}{3} + 2t,\ y = -2t,\ z = 1 + t$ intersects the plane $3x + 2y + 6z = 6$.

18 Find N for the helix given by $r(t) = (a\cos t)\,i + (a\sin t)\,\bar{j} + b\,t\,\bar{k},\ a, b \geq 0,\ a^2 + b^2 \neq 0$.

19 Find the work done by the force $F = 3yi + 2xj + 4z\,k$ along the straight line path $r(t) = ti + tj + tk, 0 \leq t \leq 1$.

20 Show that $F = (2x - 3)\,i - 2j + (\cos z)\,k$ is not conservative.

21 State normal form of Green's divergence theorem.

22 Remove the second term from the equation $x^3 - 6x^2 + 4x - 7 = 0$.

23 Form an equation whose roots are 3 times those of the equation $2x^3 - 5x^2 + 7 = 0$.

24 Find a real root of the equation $f(x) = x^3 - x - 1 = 0$ by bisection method.

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Find the curvature for the helix $r(t) = (a\cos t)\,i + (a\sin t)\,j + b + k, a, b \geq 0,\ a^2 + b^2 \neq 0$.

26 Find the mass of a wire that lies along the curve $r(t) = (t^2 - 1)\,j + 2\,tk,\ 0 \leq t \leq 1$, if the density is $\delta = \left(\frac{3}{2}\right)t$.

27 Find the flux of $F = yzj + z^2k$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1,\ Z \geq 0$ by the planes $x = 0$ and $x = 1$.

28 If α, β, γ are the roots of $x^3 + qx + r = 0$, find the equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

29 Solve by Cavdan's method $2x^3 + 3x^2 + 3x + 1 = 0$.

30 Find the root of the equation $2x = \cos x + 3$ correct to 3 decimal places by iteration method.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight of 4.

31 Use Stoke's theorem to evaluate $\int_C F \cdot dr$ if $F = xzi + xyj + 3xzk$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, transversed counter clockwise as viewed from above.

32 Solve the equation $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$ by Ferrari's method.

33 Obtain to *four* decimal places the root between 1 and 2 of the equation $x^3 - 2x^2 + 3x - 5 = 0$ by Regula-Falsi method.

$(2 \times 4 = 8)$